

Time Reversal and the CCD Matrix Decomposition

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March 25, 2004
Quantum Information Theory & Practice Seminar

National Institute of Standards and Technology

Key Ideas

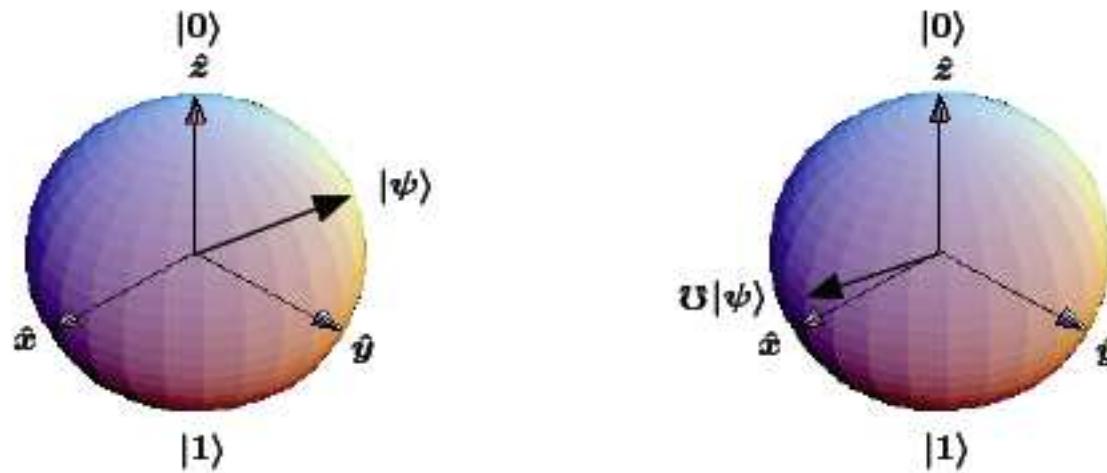
- (Time reversal and CCD) \implies Kramers' degeneracy
- Most computations: arbitrarily entangling w.r.t. concurrence
 - Even qubits: define concurrence capacity
 - Odd qubits: fun math tricks, then study $v \otimes I_2$
- (Hamiltonian H with time-reversal symmetry) \implies degenerate or highly entangled eigenstates

Outline

- I. Time Reversal, Quantum Bit-Flips, and Concurrence
- II. Concurrence Canonical Decompositons
- III. Concurrence Capacities
- IV. Algorithms Computing CCD's
- V. Concurrence of Eigenstates

Quantum Bit-Flip \mathcal{U} : One Qubit

- Picture: reverse **Bloch sphere** vector
- Bloch sphere: $\{ [|\psi\rangle] ; |\psi\rangle \in \mathcal{H}_1 \} \in \mathbb{CP}^1 = \mathbb{C}^2 / (v \sim r e^{it} v)$



Quantum Bit-Flip \mathcal{U} : One Qubit Cont.

- One qubit formula: $|\psi\rangle \xrightarrow{\mathcal{U}} (-i\sigma^y)|\overline{\psi}\rangle$
- \mathbb{C} -antilinear: not unitary evolution
- Interpretation: $\mathcal{H}_1 = \mathbb{C}|\downarrow\rangle \oplus \mathbb{C}|\uparrow\rangle$
 - Compute Bloch coordinates
 - Reverses vector on Bloch sphere: quantum angular momentum (spin) reversal
 - Physical interpretation: time-reversal symmetry operator

Bit-Flip for n -qubits

- n -qubit formula: $|\psi\rangle \xrightarrow{\mathcal{U}} (-i\sigma^y)^{\otimes n} |\overline{\psi}\rangle = \overline{(-i\sigma^y)^{\otimes n} |\psi\rangle}$
- \mathbb{C} -antilinear: **not unitary evolution**
- bit-flip \mathcal{U} : time-reversal symmetry operator
 - antiunitary (\mathbb{C} -antilinear, orthogonal)
 - $\mathcal{U}^2 = I$, n even; $\mathcal{U}^2 = -I$, n odd
- Bloch sphere flip: $|\psi\rangle = \otimes_{j=1}^n |\psi_j\rangle \implies \left(\langle \psi | \mathcal{U} | \psi \rangle = 0 \right)$

Bit-Flip and Concurrence Entanglement Monotone

- Concurrence form: $C_n(|\phi\rangle, |\psi\rangle) = \overline{\langle\phi|}(-i\sigma^y)^{\otimes n}|\psi\rangle = \overline{\langle\phi|\mathcal{U}|\psi\rangle}$
 - Overline: \mathbb{C} -linear, both variables
 - Fact: Invariant under diagonal action of LU = $\otimes_1^n SU(2)$
- Concurrence monotone: $C_n(|\psi\rangle) = |C_n(|\psi\rangle, |\psi\rangle)| = |\langle\psi|\mathcal{U}|\psi\rangle|$
- Concurrence monotone: component in quantum bit-flip

Concurrence Entanglement Monotone: Examples

- $C_{2p-1}(|\psi\rangle) = 0$, for any $|\psi\rangle$
- $|\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)$, then $C_4(|\text{GHZ}\rangle) = 1$
- $|W\rangle = \frac{1}{2}(|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle)$, then $C_4(|W\rangle) = 0$
- $C_{2p}(|\psi_{2p-1}\rangle \otimes |\psi_1\rangle) = 0$, in particular vanishes on locals

$$C_n(|\phi\rangle, |\psi\rangle) = C_n(v|\phi\rangle, v|\psi\rangle), \text{any } v \text{ in LU}$$

Proof: Proceed as follows.

- For 2×2 matrix m , note that $\mathbf{m}^T(-i\sigma^y)m = (\det m)(-i\sigma^y)$
 - $v^T(-i\sigma^y)w$ is \pm area spanned by v, w
 - Check equation by considering columns, rows of m
- Let $v = v_1 \otimes v_2 \otimes \cdots \otimes v_n$; each $\det v_j = 1$ by choice global phase
- $(v_1 \otimes v_2 \otimes \cdots \otimes v_n)^T(-i\sigma^y)^{\otimes n}(v_1 \otimes v_2 \otimes \cdots \otimes v_n) = \otimes_{j=1}^n [v_j^T(-i\sigma^y)v_j] = (-i\sigma^y)^{\otimes n}$

Remark: Similar proof shows $C_n(|\psi_{n-1}\rangle \otimes |\phi_1\rangle) = 0$.

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Natural: CCD by $G = KAK$ metadecomposition

- Metadecomposition: theorem outputting matrix decompositions
- Three cascading inputs, each dependent on last
- Generalizes canonical dec.: $SU(4) = [SU(2) \otimes SU(2)]\Delta[SU(2) \otimes SU(2)]$
 - $SU(2) \otimes SU(2)$: two-qubit local unitary (LU) group
 - Δ : relative phase computations on Bell (or “magic”) basis
- Sample applications: 2q control theory, 2q quantum logic circuits, 2q computation times, 2q entanglement theory

Cascading Inputs of $G = KAK$ theorem

1. Lie group G , semisimple, $\mathfrak{g} = \text{Lie}(G)$ ($\implies G = \exp \mathfrak{g}$, matrix exponential)
2. Cartan involution $\theta : \mathfrak{g} \rightarrow \mathfrak{g}$ (using same term if G compact)
 - $[\theta X, \theta Y] = \theta[X, Y]$
 - $\theta^2 = I_N$ (involution)
 - Notation: $\mathfrak{g} = \mathfrak{p} \oplus \mathfrak{k}$, where $\mathfrak{p} = \mathfrak{g}_{-1}$, $\mathfrak{k} = \mathfrak{g}_{+1}$
 - Encodes generalized polar decomp. of \mathfrak{g}
3. (Maximal-Commutative) subspace $\mathfrak{a} \subset \mathfrak{p}$

$G = KAK$ Example: Bloch Sphere Rotations

- Take $G = SU(2)$ (prefix SG : determinant-one subgroup)
- $\mathfrak{su}(2) = \{A = iH \in \mathbb{C}^{2 \times 2} ; A^\dagger = -A, \text{tr } A = 0\}$
- $\theta(X) = -X^T = \overline{X}$, fixes $\mathfrak{k} = \mathfrak{so}(2) = \mathfrak{su}(2) \cap \mathbb{R}^{2 \times 2}$
- $\{R_y(t)\} = SO(2) = \exp \mathfrak{so}(2)$, the Y -axis Bloch sphere rotations
- Take $\mathfrak{a} = \mathbb{R}(i|0\rangle\langle 0| - i|1\rangle\langle 1|)$, so $A = \{R_z(t)\}$
- Result: $SU(2) = \{R_y(t)\}\{R_z(t)\}\{R_y(t)\}$, the **YZY -decomposition**

$G = KAK$ Example: 2q Canonical Decomposition

- $G = SU(4)$, with $\mathfrak{g} = \mathfrak{su}(4) = \{X = iH \in \mathbb{C}^{4 \times 4} ; H = H^\dagger, \text{tr } H = 0\}$
- $\theta(X) = (-i\sigma^y)^{\otimes 2}[-X^T](-i\sigma^y)^{\otimes 2} = (-i\sigma^y)^{\otimes 2}[\bar{X}](-i\sigma^y)^{\otimes 2}$
- Check: for this θ , in fact $K = SU(2) \otimes SU(2)$
 - +1 eigenspace of θ is $[I_2 \otimes \mathfrak{su}(2)] \oplus [\mathfrak{su}(2) \otimes I_2]$
 - Product Rule: $\text{Lie}[SU(2) \otimes SU(2)] = [I_2 \otimes \mathfrak{su}(2)] \oplus [\mathfrak{su}(2) \otimes I_2]$
- May choose $A = \Delta$ phasing Bell basis so that $\mathfrak{a} \subset \mathfrak{p}$
- Result: canonical dec. $SU(4) = [SU(2) \otimes SU(2)]\Delta[SU(2) \otimes SU(2)]$

CCD: Extend $G = KAK$ Inputs

- Extends Euler angle and $2q$ canonical decomp. examples
- $G = KAK$ Input #1: $G = SU(2^n)$
- $G = KAK$ Input #2: $\theta(iH) = [(-i\sigma^y)^{\otimes n}]^\dagger \overline{(iH)} (-i\sigma^y)^{\otimes n} = \mathcal{U}(iH)\mathcal{U}^{-1}$
- \mathfrak{k} is $+1$ -eigenspace; $K = \exp(\mathfrak{k})$
- \mathfrak{a} : parity-dependent subalgebra of \mathfrak{p} ; $A = \exp(\mathfrak{a})$
- Concurrence Canonical Decomposition is $SU(N) = KAK$

CCD and Time-Reversal Symmetry

- Concurrence Cartan Involution: $\theta(iH) = [(-i\sigma^y)^{\otimes n}]^\dagger \overline{(iH)} (-i\sigma^y)^{\otimes n} = \mathcal{U}(iH)\mathcal{U}^{-1}$
 - $\mathfrak{su}(N) = \mathfrak{p} \oplus \mathfrak{k}$, -1 and $+1$ eigenspace respectively
 - ($iH \in \mathfrak{p}$) \Rightarrow (H has \mathcal{U} time-reversal symmetry)
 - ($iH \in \mathfrak{k}$) \Rightarrow (H has \mathcal{U} time-reversal anti-symmetry)
- Fact: Always $kak^\dagger \in \exp(\mathfrak{p})$, given $k \in K$, $a \in A$
- **CCD application:** any $v = k_1ak_2 = (k_1ak_1^\dagger)(k_1k_2) = \exp(iH_{\mathfrak{p}})\exp(iH_{\mathfrak{k}})$

CCD and Pauli-Operator Basis

- Observed slightly earlier: Bremner-Dodd-Nielsen-Bacon
- Notation: $i\sigma^{\otimes J}$, multiindex $J = j_1 j_2 \cdots j_k \cdots j_n$
 - $\sigma^0 = I$, $j \in \{0, x, y, z\}$
 - $i\sigma^{\otimes J} = i \otimes_{j=1}^n \sigma^j \in \mathfrak{u}(N)$
 - Traceless if some $j_k \neq 0$; $\#J = \#\{k ; j_k \neq 0, 1 \leq k \leq n\}$
- $\mathfrak{su}(N) = \{ iH ; H = H^\dagger, \text{tr}(H) = 0 \} = \bigoplus_{\#J \neq 0} \mathbb{R} i\sigma^{\otimes J}$

CCD and Pauli-Operator Basis Cont.

- \mathbb{F}_2 grading: $\mathfrak{su}(N) = \left(\bigoplus_{\#J \equiv 0 \pmod{2}, \#J \neq 0} \mathbb{R}\{i\sigma^{\otimes J}\} \right) \oplus \left(\bigoplus_{\#J \equiv 1 \pmod{2}} \mathbb{R}\{i\sigma^{\otimes J}\} \right)$
- time-reversal symmetry w.r.t. \mathcal{U} : $\mathfrak{p} = \bigoplus_{\#J \equiv 0, \pmod{2}, \#J \neq 0} \mathbb{R} i\sigma^{\otimes J}$
- time-reversal anti-symmetry w.r.t. \mathcal{U} : $\mathfrak{k} = \bigoplus_{\#J \equiv 1 \pmod{2}} \mathbb{R} i\sigma^{\otimes J}$
- **Preview:** Demonstrate eigenstates of all $iH \in \mathfrak{p}$ are degenerate or else have concurrence one

Interpretation: K Fixes Concurrence

Theorem (—,GKB): Let $K = \exp \mathfrak{k}$ for \mathfrak{k} the $+1$ -eigenspace of the concurrence Cartan involution $\theta(iH) = [(-i\sigma^y)^{\otimes n}]^\dagger \overline{(iH)} (-i\sigma^y)^{\otimes n}$. Then K is the symmetry group of the concurrence form $C_n(-, -)$. Specifically, for $v \in SU(N)$,

$$(v \in K) \iff [C_n(v|\phi\rangle, v|\psi\rangle) = C_n(|\phi\rangle, |\psi\rangle) \quad \text{for every } |\phi\rangle, |\psi\rangle \in \mathcal{H}_n]$$

Moreover, for $C_n(y, x) = (-1)^n C_n(x, y)$, and auxilliary constructions demand

- abstract isomorphism $K \cong Sp(N/2)$, a symplectic group, $n = 2p - 1$
- abstract isomorphism $K \cong SO(N)$, $n = 2p$

Generalized Kramers' Degeneracy and A

- **Kramers' degeneracy:** Half integral spin system, time-symmetric energy Hamiltonian \implies degenerate eigenstates
- Time-symmetric Hamiltonian: CCD simplifies
 - $v = \exp(iH)$, $v = kak^\dagger$ and $H = kH_\mathfrak{a}k^\dagger$, $H_\mathfrak{a} \in \mathfrak{a}$
- Structure of \mathfrak{a} algebra (depends on parity of n)
 - n (integral spin), no repeat eigenvalues
 - $n = 2p - 1$, **repeat eigenvalues** \implies degeneracy **implicit in math**

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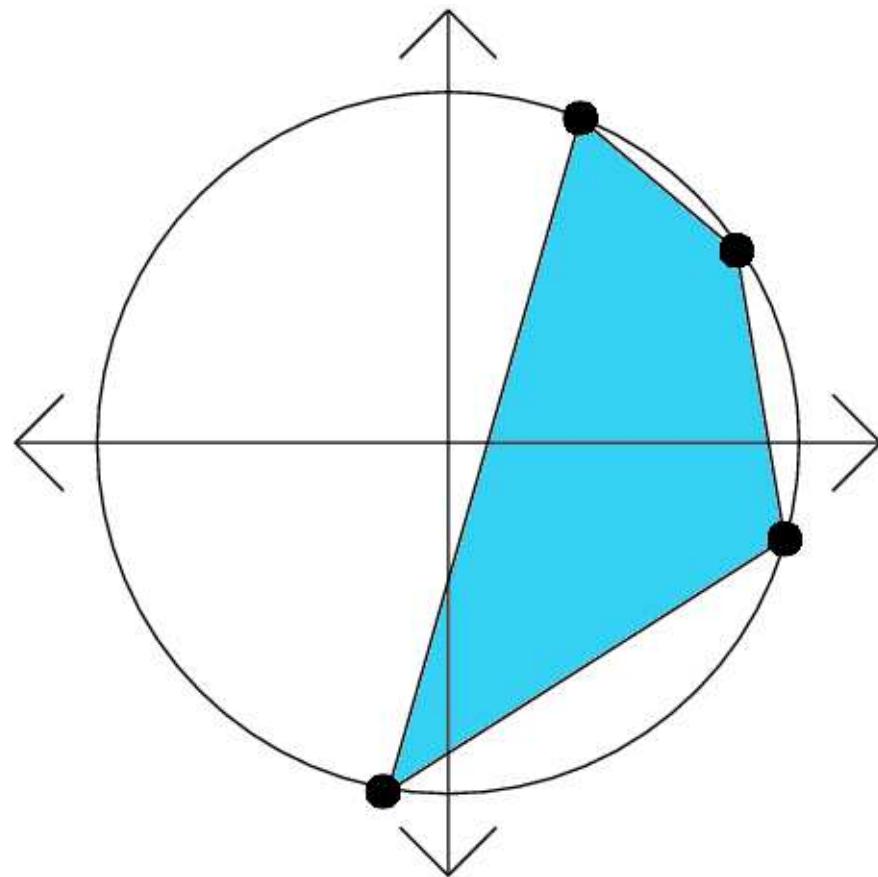
Two-Qubit Entanglement Capacities

- Zhang et al (Berkeley): two-qubit entanglement capacities
 - Def: $\mathcal{E}_2(\nu) = \max\{ C_2(\nu|\psi\rangle) ; C_2(|\psi\rangle) = 0 \}$
 - Strategy: study changes in entanglement induced by a factor of two-qubit canonical decomp, $\nu = [b \otimes c]a[d \otimes f]$
 - Thm: $\mathcal{E}_2(\nu) = 1$ iff the convex hull (polygonal span) of $\text{spec}[(\sigma^y)^{\otimes 2} \nu (\sigma^y)^{\otimes 2} \nu^T]$ holds $0 \in \mathbb{C}$
- Application: B gate; two-qubit computation with minimal possible (2) applications needed to build $SU(4)$ from $\{B, LU\}$

Even Qubit Concurrence Capacities

- Fix $n = 2p$, integral spin qubit system, $K \cong SO(N)$
 - Def: Concurrence capacity
$$\tilde{\kappa}_{2p}(v) = \max\{ C_{2p}(v|\psi\rangle) ; C_{2p}(|\psi\rangle) = 0, \langle\psi|\psi\rangle = 1 \}$$
 - Prop:(BB) $\tilde{\kappa}_{2p}(k_1 a k_2) = \tilde{\kappa}_{2p}(a)$
 - Thm:(BB) $\tilde{\kappa}_{2p}(v) = 1$ iff the convex hull (polygonal span) of $\text{spec}[(-i\sigma^y)^{\otimes 2p} v (-i\sigma^y)^{\otimes 2p} v^T] = \text{spec}(a^2)$ holds $0 \in \mathbb{C}$
- Application: Choose an element a at random for $2p$ large \implies the concurrence capacity of $v = k_1 a k_2$ is probably one

Picture: Sample Convex Hull



Odd Qubit Capacities??

- **Problem:** $C_{2p-1}(|\psi\rangle) = 0$, every $|\psi\rangle \in \mathcal{H}_{2p-1}$
- **Work-around:** Pairwise capacity (normalized kets):

$$\kappa_n(v) = \max\{ C_n(v|\phi\rangle, v|\psi\rangle) ; C_n(|\phi\rangle, |\psi\rangle) = 0 \}$$

- **Properties of pairwise capacity**
 - $\kappa_{2p}(v) = \tilde{\kappa}_{2p}(v)$ yet $\kappa_{2p-1} \not\equiv 0$
 - $\kappa_{n+1}(v \otimes I_2) \geq \kappa_n(v)$, i.e. large pairwise capacity for $n = 2p - 1$ implies $v \otimes I_2$ entangles

Odd Qubit Capacities?? Cont.

Theorem: For $n = 2p$ or $n = 2p - 1$, label a concurrence spectrum of $v \in SU(2^n)$ by $\lambda_c(v) = \text{spec} \left([(-i\sigma^y)^{\otimes n}]^\dagger v (-i\sigma^y)^{\otimes n} v^T \right)$.

- $\lambda_c(v) = \text{spec}(a^2)$ for any CCD $v = k_1 \circ k_2$
- $\left(\kappa_n(v) = 1 \right) \iff \left(0 \in \text{convex hull}[\lambda_c(v)] \right)$
- **Remark:** Second item can be understood directly for $n = 2p - 1$
- **Again:** Most unitaries produce full concurrence

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Algorithm for CCD, $n = 2p$

- **Similarity matrix E :** columns all fully concurrent
 - Translates CCD to $SU(2^{2p}) = SO(2^{2p}) D SO(2^{2p})$
 - $v \in SU(2^{2p}), EvE^\dagger = o_1 d o_2 \implies$ CCD $v = (Eo_1E^\dagger)(EdE^\dagger)(Eo_2E^\dagger)$
- **Question:** How to compute $v = o_1 d o_2$?
- **STEP #1:** Compute $o_1 d^2 o_1^\dagger = vv^T$

Algorithm for CCD, $n = 2p$ Cont.

- **STEP #2:** Diagonalize $p = vv^T = o_1 d^2 o_1^\dagger$ over real kets
 - $p = a + ib$, $p = p^T \implies a = a^\dagger$, $b = b^\dagger$
 - $pp^\dagger = I_{2^n} \implies [a, b] = \mathbf{0}$
 - So diagonalize a, b over same o_1
- **STEP #3:** Take $\sqrt{p} = o_1 d o_1^\dagger$; back-solve for o_2

$n = 2p - 1$ **Case:** $K \cong Sp(2^n/2)$

- $Sp(2^{2p-1}/2)$: Symmetries of $\mathcal{A}(|\phi\rangle, |\psi\rangle) = \overline{\langle \phi |} J |\psi\rangle$, $J = (-i\sigma^y) \otimes I_{N/2}$
 - $\mathcal{A}(-, -)$ is **antisymmetric**
 - $Sp(N/2)$ time-reversal antisymmetric evolutions for $|\psi\rangle \xrightarrow{\Theta} J \overline{|\psi\rangle}$
- Dyson: quaternion formulation
- Block form of Lie symmetry group:

$$Sp(2^{2p-2}) = \left\{ \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in SU(N); \begin{array}{l} A^T C \text{ is symmetric}, B^T D \text{ is symmetric}, \\ A^T D - C^T B = I_{2p-2} \end{array} \right\}$$

Algorithm for CCD, $n = 2p - 1$

- **Similarity matrix F :** columns all fully concurrent
 - CCD $\xrightarrow{\text{Ad}(F)} SU(2^{2p}) = Sp(2^{2p-1}/2) D Sp(2^{2p-1}/2)$, D repeat diagonal
 - $v \in SU(2^{2p})$, $FvF^\dagger = \omega_1 (d \otimes I_2) \omega_2 \implies \text{CCD } v = (F\omega_1 F^\dagger)(Fd \otimes I_2 F^\dagger)(F\omega_2 F^\dagger)$
- **Question:** How to compute $v = \omega_1 d \otimes I_2 \omega_2$?
- **STEP #1:** Compute $p = \omega_1 (d^2 \otimes I_2) \omega_1^\dagger = -v J v^T J$

Algorithm for CCD, $n = 2p - 1$ Cont.

- **STEP #2:** Diagonalize $H = \frac{i}{2} \log p$ over some $\omega \in Sp(N/2)$
 - **Complicated:** Dongarra, Gabriel, Koelling, Wilkinson
Linear Algebra & Applications, 1984
 - $\Theta|\psi\rangle = [(-i\sigma^y) \otimes I_{N/2}]|\overline{\psi}\rangle$: H has Θ time-reversal symmetry
- **STEP #3:** Take $p = \omega_1(d \otimes I_2)\omega_1^\dagger$; back-solve for ω_2

Symplectic Diagonalization in Practice

- Input time-symmetric H must have block form
 - Theory: $H = \frac{i}{2} \log p$, $p = -vJv^T J$ always has block form
 - Computation: H miraculously has block form
- View H in 2×2 blocks; diagonalize each
- Diagonalization Preserves Symplectic Structure:
 - $|\psi\rangle$ is λ -eigenstate \implies Θ -momentum reversal $[(-i\sigma^y) \otimes I_2]|\overline{\psi}\rangle$ also
 - Columns of ω preserve property; each substep

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Eigenstates for Time-Symmetric H

Prop (Kramers): Let H be a traceless Hamiltonian which is time-reversal symmetric (i.e. $iH \in \mathfrak{su}(N)$, $iH \in \mathfrak{p}$.) Say $|\psi\rangle \in \mathcal{H}_n$ is a λ -eigenstate. Then the bit-flip $\mathcal{U}|\psi\rangle$ is also a λ -eigenstate.

Proof: Since $H = H^\dagger$, note that λ is real. Thus antilinearity causes $\mathcal{U}\lambda|\psi\rangle = \lambda\mathcal{U}|\psi\rangle$. By symmetry, $\mathcal{U}H\mathcal{U}^{-1} = H$, i.e. $\mathcal{U}H = H\mathcal{U}$. Thus given $H|\psi\rangle = \lambda|\psi\rangle$,

$$H\mathcal{U}|\psi\rangle = \mathcal{U}H|\psi\rangle = \mathcal{U}\lambda|\psi\rangle = \lambda\mathcal{U}|\psi\rangle$$

Thus $\mathcal{U}|\psi\rangle$ is a λ -eigenstate. □

Derivation of Kramers' Degeneracy

- Fix $n = 2p - 1$, H traceless, time-reversal symmetric
- $\mathcal{C}_{2p-1}(|\psi\rangle, |\psi\rangle) = 0$, and $|\psi\rangle$
 - $(-i\sigma^y)^{\otimes 2p-1} = -[(-i\sigma^y)^{\otimes 2p-1}]^T$
 - Hence $\mathcal{C}_{2p-1}(|\phi\rangle, |\psi\rangle) = -\mathcal{C}_{2p-1}(|\psi\rangle, |\phi\rangle)$
- $\overline{\mathcal{C}_{2p-1}(|\psi\rangle, |\psi\rangle)} = \langle \psi | \mathcal{U} | \psi \rangle = 0$, any $|\psi\rangle$ (odd case)
- Hence all eigenstates of H are degenerate!
Eigenspace holds $|\lambda\rangle$ and Hermitian-orthonormal $\mathcal{U}|\lambda\rangle$

Highly Entangled Eigenstates

- Fix $n = 2p$, H traceless, time-symmetric, nondegenerate
- $H|\psi\rangle = \lambda|\psi\rangle \implies U|\psi\rangle$ also λ -eigenstate
- Nondegenerate: must have $U|\psi\rangle = e^{i\phi}|\psi\rangle$
- Consequence: $C_{2p}(|\psi\rangle) = |\langle\psi|U|\psi\rangle| = 1$;
any nondegenerate eigenstate of H is maximally concurrent
(\implies very entangled)

Examples of Time-Symmetric Hamiltonians

- E.g. XY Hamiltonian: $H_{XY} = \sum_{j=1}^{2p-1} \frac{1+g}{4} \sigma_j^x \sigma_{j+1}^x + \frac{1-g}{4} \sigma_j^y \sigma_{j+1}^y$
- Time symmetric: Each summand has even # of Pauli operators.
- Literature: **nondegenerate** (no repeat eigenvalues)
- Consequence: **Very entangled eigenstates**, in particular ground state
- Could one produce maximally concurrent states by **cooling**?

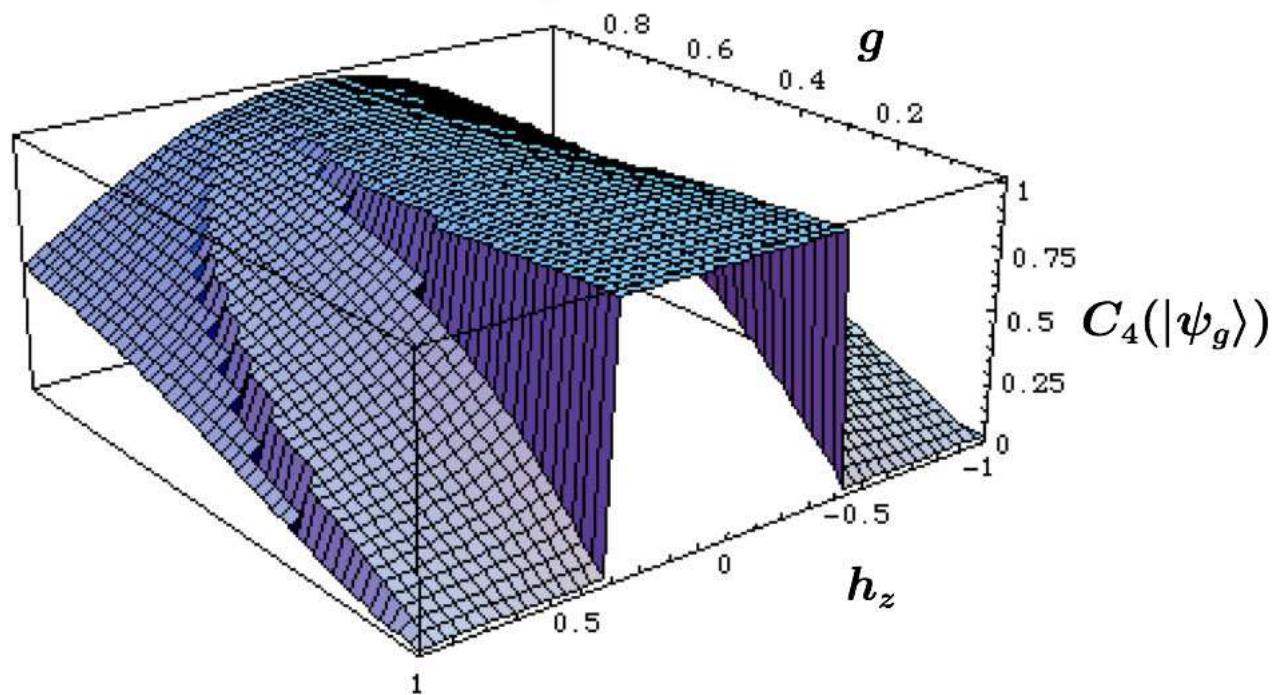
Perturbations of Time Symmetry (GKB)

- Proof fails if perturbative individual one-qubit spins are added:

$$H = \sum_{j=1}^{2p-1} \left(\frac{1+g}{4} \sigma_j^x \sigma_{j+1}^x + \frac{1-g}{4} \sigma_j^y \sigma_{j+1}^y \right) + \frac{h_z}{2} \sum_{j=1}^{2p-1} \sigma_j^z$$

- **Question (GKB):** How does concurrence change in h_z and in balance of $\sigma^x \otimes \sigma^x$ vs. $\sigma^y \otimes \sigma^y$ given by g ?
- **Four qubit answer:** over

Perturbations of Time Symmetry (GKB) Cont.



Key Ideas, Reconsidered

- (Time reversal and CCD) \implies Kramers' degeneracy
- Most computations: arbitrarily entangling w.r.t. concurrence
 - Warning! $C_{2p}(|\psi\rangle) = 1$ is much weaker than being in SLOCC orbit of $|\text{GHZ}\rangle$!
 - Interest (Scott,Caves): Most states have $C_{2p}(|\psi\rangle)^2$ small.
- (Hamiltonian H with time-reversal symmetry) \implies degenerate or highly entangled eigenstates, e.g. groundstates

Ongoing Work

- More numerics on “concurrence phase-transition”(??)

- Concurrence one states

$$- \left\{ |\psi\rangle ; \langle \psi | \psi \rangle = 1, C_{2p}(|\psi\rangle) = 1 \right\} = \left\{ k|GHZ\rangle ; k \in K \right\}$$

- dim K exponential, but smaller than $SU(2^{2p})$
 - Are such $|\psi\rangle$ useful?? Implement K ??
-
- Apply CCD to quantum-logic circuit design